EXCITATION OF METASTABILE STATES IN GASEOUS NEBULAE

There are two different types of atomic processes which are responsible for the excitation of metastabile states in the gaseous nebulae: the fluorescence phenomenon and the electronic collisions.

The fluorescence phenomenon. We consider an atom which has three stationary levels 1, 2, 3 with the enrgies $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$. Let n_1, n_2, n_3 be the number of atoms in cubic centimetre in corresponding levels. The relative values of these numbers in the case of stationary distribution are determined by radiation field and atomic constants (transition probabilities). The stationarity conditions are:

$$\begin{aligned} &B_{12}\left(n_1 - \frac{g_1}{g_2}\cdot n_2\right)\rho_{12} + B_{13}\left(n_1 - \frac{g_1}{g_3}\cdot n_3\right) - n_2\frac{g_1}{g_2}\,B_{12}\,\sigma_{12} - n_3\frac{g_1}{g_3}\,B_{13}\,\sigma_{13} = 0 \\ &B_{13}\left(n_1 - \frac{g_1}{g_3}\cdot n_3\right)\rho_{13} + B_{23}\left(n_2 - \frac{g_2}{g_3}\cdot n_3\right)\rho_{23} - n_3\left\{\frac{g_1}{g_3}\,B_{13}\,\sigma_{13} + \frac{g_2}{g_3}\,\sigma_{23}\,B_{23}\right\} = 0, \end{aligned} \tag{1}$$

where B_{ik} is Einstein's probability coefficient corresponding to the transition $i \to k$, g_k is the weight of the k–th level, ρ_{ik} is the density of radiation in the frequency

$$\nu_{ik} = \frac{\varepsilon_k - \varepsilon_i}{h}$$

and

$$\sigma_{ik} = \frac{8\pi\, h\nu_{ik}^3}{c^3}, \tag{2}$$

h,c and π have their usual meaning.
Thus the product

$$A_{ki} = B_{ik}\,\sigma_{ik}\frac{g_i}{g_k} \tag{3}$$

is the Einstein's probability coefficient of spontaneous transition $k \to i$.
We write the equations (1) to become

$$\begin{aligned} &B_{12}\,\rho_{12} + B_{13}\,\rho_{13} = \frac{g_1}{g_2}\,B_{12}\,(\sigma_{12} + \rho_{12})\,\frac{n_2}{n_1} + \frac{g_1}{g_3}\,B_{13}\,(\sigma_{13} + \rho_{13})\,\frac{n_3}{n_1} \\ &B_{13}\,\rho_{13} = -B_{23}\,\rho_{23}\frac{n_2}{n_1} + \left[\frac{g_1}{g_3}\,B_{13}\,(\sigma_{13} + \rho_{13}) + \frac{g_2}{g_3}\,B_{23}\,(\sigma_{23} + \rho_{23})\right]\frac{n_3}{n_1}. \end{aligned} \tag{4}$$

Before solving these equations we make some simplifications, corresponding to the physical conditions in nebulae. The radiation density ρ_{ik} may be represented as

$$\rho_{ik} = W\frac{\sigma_{ik}}{\exp\left(\frac{h\nu_{ik}}{k\,T}\right) - 1}. \tag{5}$$

Here T is the surface temperature of the nucleus and the factor W is defined by the relation:

$$W = \frac{1}{4}\left(\frac{r_*}{r_n}\right)^2, \tag{6}$$

where r_* is the radius of the nucleus and r_n is the distance of the point of nebula we consider from the nucleus. If W is a small quantity ($W < 10^{-3}$) the densities ρ_{ik} in the brackets of (4) may be neglected, compared with σ_{ik} and we have:

$$\begin{aligned} B_{12}\,\rho_{12} + B_{13}\,\rho_{13} &= \frac{g_1}{g_2}\,B_{12}\,\sigma_{12}\,\frac{n_2}{n_1} + \frac{g_1}{g_3}\,B_{13}\,\sigma_{13}\,\frac{n_3}{n_1} \\ B_{13}\,\rho_{13} &= -B_{23}\,\rho_{23}\frac{n_2}{n_1} + \left(\frac{g_1}{g_3}\,B_{13}\,\sigma_{13} + \frac{g_2}{g_3}\,B_{23}\,\sigma_{23}\right)\frac{n_3}{n_1}. \end{aligned} \tag{7}$$

Solving these equations we obtain:

$$\frac{n_2}{n_1} = \frac{B_{12}\,\rho_{12}\,(g_1 B_{13}\sigma_{13} + g_2 B_{23}\sigma_{23}) + g_2\,B_{13}B_{23}\rho_{13}\sigma_{23}}{\frac{g_1}{g_2}\,B_{12}\,\sigma_{12}\,(g_1 B_{13}\sigma_{13} + g_2 B_{23}\sigma_{23}) + g_1\,B_{13}B_{23}\sigma_{13}\rho_{23}}. \tag{8}$$

We suppose that the second level is a metastabile one, i.e. that the quantity B_{12} is small compared with B_{13} and B_{23}. Therefore the members containing the factor $B_{12}\,\rho_{12}$ may be neglected compared with the term containing $B_{13}\,\rho_{13}$. (8) then becomes:

$$\frac{n_2}{n_1} = \frac{g_2\,B_{13}B_{23}\rho_{13}\sigma_{23}}{\frac{g_1}{g_2}\,B_{12}\,\sigma_{12}\,(g_1 B_{13}\sigma_{13} + g_2 B_{23}\sigma_{23}) + g_1\,B_{13}B_{23}\sigma_{13}\rho_{23}}. \tag{9}$$

We may write

$$\rho_{ik} = W\sigma_{ik}\,\overline{\rho}_{ik} \quad \text{where} \quad \overline{\rho}_{ik} = \frac{1}{\exp\left(\frac{h\nu_{ik}}{kT}\right) - 1}. \tag{10}$$

Then

$$\frac{n_2}{n_1} = \frac{g_2\overline{\rho}_{13}\,W}{\frac{g_1^2}{g_2}\,\frac{B_{12}}{B_{23}}\,\frac{\sigma_{12}}{\sigma_{23}} + g_1\,\frac{B_{12}}{B_{13}}\,\frac{\sigma_{12}}{\sigma_{13}} + g_1\overline{\rho}_{23}\,W}. \tag{11}$$

Neither the transition $3 \to 1$, nor the transition $3 \to 2$ are forbidden. Therefore the quantities B_{13} and B_{23} will be of the same order of magnitude. Therefore the ratios $\frac{B_{12}}{B_{13}}$ and $\frac{B_{12}}{B_{23}}$ are small quantities of the same order of magnitude.

Case I. $W < \frac{B_{12}}{B_{13}}, \frac{B_{12}}{B_{23}}$. In this case the last term in denominator may be neglected. ($\overline{\rho}_{23}$ is ordinarily of the order of unity) and we have

$$\frac{n_2}{n_1} = \frac{g_2}{g_1}\,W\frac{\overline{\rho}_{13}}{\frac{g_1}{g_2}\,\frac{B_{12}}{B_{23}}\,\frac{\sigma_{12}}{\sigma_{23}} + \frac{B_{12}}{B_{13}}\,\frac{\sigma_{12}}{\sigma_{13}}}.$$

In order to estimate the order of magnitude of $\frac{n_2}{n_1}$ we may put: $g_1 = g_2$; $B_{23}\,\sigma_{23} = B_{13}\,\sigma_{13}$. We get

$$\frac{n_2}{n_1} \cong W\frac{B_{13}\,\sigma_{13}}{2\,B_{12}\,\sigma_{12}}\cdot\overline{\rho}_{13}. \tag{12}$$

If on the other hand the second level is not metastabile (ordinary level) and B_{12} is of the same order of magnitude as B_{13} and B_{23}, we may neglect the last term in denominator of (8) and write

$$\left(\frac{n_2}{n_1}\right)_{\rm ord} = \frac{B_{12}\,\sigma_{12}\,(g_1 B_{13}\sigma_{13} + g_2 B_{23}\sigma_{23})\; W\overline{\rho}_{12} + g_2\, B_{13}B_{23}\sigma_{13}\sigma_{23}\, W\overline{\rho}_{13}}{\frac{g_1}{g_2}\, B_{12}\,\sigma_{12}\,(g_1 B_{13}\sigma_{13} + g_2 B_{23}\sigma_{23})}. \qquad (13)$$

When estimating the order of magnitude we may put: $g_1 = g_2$; $B_{12}\,\sigma_{12} = B_{13}\,\sigma_{13} = B_{23}\,\sigma_{13}$. Then

$$\left(\frac{n_2}{n_1}\right) = W\overline{\rho}_{12} + \frac{1}{2}\,W\,\overline{\rho}_{13}. \qquad (14)$$

The main difference between (12) and (14) is the presence in (12) of a large factor $\frac{B_{13}\,\sigma_{13}}{B_{12}\,\sigma_{12}}$. Therefore we may assert that in our case the number of atoms in the metastabile state is $\frac{B_{13}\,\sigma_{13}}{B_{12}\,\sigma_{12}}$ times larger than in any ordinary excited state.

Case II. $W > \frac{B_{12}}{B_{13}},\ \frac{B_{12}}{B_{23}}$. In this case the first two numbers in the denominator of (11) may be neglected. Therefore we have

$$\frac{n_2}{n_1} = \frac{g_2}{g_1}\,\frac{\overline{\rho}_{13}}{\overline{\rho}_{23}} = \frac{g_2}{g_1}\,\frac{\exp\left(\frac{h\nu_{23}}{k\,T}\right) - 1}{\exp\left(\frac{h\nu_{13}}{k\,T}\right) - 1}. \qquad (15)$$

If $h\,\nu_{13} > k\,T$ we obtain

$$\frac{n_2}{n_1} = \frac{g_2}{g_1}\left(\exp\left(-\frac{h\nu_{12}}{k\,T}\right) - \exp\left(-\frac{h\nu_{13}}{k\,T}\right)\right). \qquad (16)$$

If at the same time $h\,\nu_{23} > k\,T$and $h\,\nu_{12} > k\,T$

$$\frac{n_2}{n_1} = \frac{g_2}{g_1}\,\exp\left(-\frac{h\nu_{12}}{k\,T}\right). \qquad (17)$$

In this case the ratio $\frac{n_2}{n_1}$ is approximately defined by Boltzmann's law. This result was already obtained by Rosseland.[1]

The physical meaning of the above is the following: The first two terms in the denominator of (11) correspond to the forbidden transition from the metastabile state to the normal state. The last term in the denominator of (11) corresponds to the transitions from the metastabile state to the higher states. These transitions are stimulated by the corresponding radiation. In the first case the forbidden transitions are predominant. The forbidden line will appear then in full strength. In the second case the stimulated transitions to the higher levels are predominant, and if W is sufficiently large, the relative number of forbitten transitions will be very small and the forbidden line will disappear. Shortly, in the second case the density of radiation will be large enough to make the collisions of the metastabile atoms with light–quanta sufficiently frequent.

[1]S. Rosseland, Astrophysica.

There may be some doubts as to the possibility of application of our formulae to the gaseous nebulae because the photo–electric ionization plays in these nebulae a far more important role than the line excitation. But we may treat the ionized atom as an atom in the energy level with very large weight g_3, and the continuous spectrum behind the head of the principal series of atom as a very wide spectral line. In fact the quantity B_{13} determined from the condition that $n_1\,B_{13}\,\rho_{13}$ is the number of atoms ionized per second, will be of the same order of magnitude as the B— coefficients for the first lines of the principal series. We remark that on account of large optical thickness of nebula in ordinary lines of the principal series the radiation of nucleus in these lines will be absorbed in the inner layers of the nebula and therefore the first member of (14) vanishes while the second member remains nearly unchanged since the optical thickness in the continuous spectrum is about 10^4 times smaller than in the ordinary lines of the principal series. Hence

$$\left(\frac{n_2}{n_1}\right)_{\rm ord} = \frac{1}{2}\cdot W\,\overline{\rho}_{13} = \frac{1}{2}\frac{W}{\exp\left(\frac{h\nu_{13}}{k\,T}\right)-1}. \tag{18}$$

when the second level is not a metastabile one.
The author's observations [1] are in good agreement with this formula. The expression (18) shows that our assertion that in case I the number of atoms in the metastabile state is $\dfrac{B_{13}\,\sigma_{13}}{B_{12}\,\sigma_{12}}$ times larger, than in any ordinary excited state must be satisfied more exactly, than expected.

Applications to the gaseous nebulae. We have

$$A_{ki} = \frac{g_i}{g_k}\cdot B_{ik}\,\sigma_{ik}.$$

For the first lines of each principal series A_{ki} is of the order 10^8 sec^{-1} if the corresponding transition is not forbidden. Taking $g_i = g_k$ we obtain for these lines

$$B_{ik} = \frac{10^8}{\sigma_{ik}}.$$

As we have mentioned above, the quantity B_{13} corresponding to the bound–free transitions will be of this order of magnitude. In the first case it will be

$$W < \frac{B_{12}}{B_{13}} \quad \text{or} \quad W < \frac{B_{12}\,\sigma_{13}}{10^8},$$

or introducing $B_{12} = \dfrac{A_{12}}{\sigma_{12}}\cdot\dfrac{g_2}{g_1}$

$$W < \frac{A_{12}}{10^8}\cdot\frac{\sigma_{13}}{\sigma_{12}}\cdot\frac{g_2}{g_1} = 10^{-8}\,\frac{g_2}{g_1}\cdot\left(\frac{\nu_{13}}{\nu_{12}}\right)^3.$$

The quantity $\dfrac{g_2}{g_1}\cdot\left(\dfrac{\nu_{13}}{\nu_{12}}\right)^3$ is usually of the order of unity and we find

$$W < 10^{-8}A_{12}.$$

In the planetaries and diffuse nebulae W is of the order 10^{-14}. Hence

$$\tau_2 = \frac{1}{A_{12}} < 10^6 \text{ sec.},$$

where τ_2 is the mean life–time of the metastabile state.

Thus, if the mean life–time of the metastabile state is shorter than a week, the conditions of Case I are fulfilled. Only when the mean life–time of the given state is larger than 10^6 sec. will the formulae of Case II be applicable. As examples we shall consider the following metastabile levels: the states 2 S of H, 2 S of $\mathrm{H}e^+$, 2^1 S of parhelium and the state $2^3\,S$ of orthohelium. The first three of these are metastabile because the only possible transition of the type

$$2\,\mathrm{S} \longrightarrow 1\,\mathrm{S}$$

is "forbidden" as a transition between two even states. The last state 2^3 S of orthohelium is metastabile because the only possible transition of the type

$$2^3\,\mathrm{S} \longrightarrow 1^1\,\mathrm{S}$$

is forbidden not only as a transition from one even state to another but also as an intercombination between an orthohelium and a parhelium levels. The metastability of 2^3 S of *He* will be therefore of a higher degree than the metastability of the first three levels.

If we suppose that the mean life–time for the first three types is of the order of 1 sec. or 10 sec. i.e. of the same order as the mean life–time of the levels corresponding to the "nebulium" radiation the formulae of Case I will be applicable. The ratio $\frac{n_2}{n_1}$ will be for these states 10^8 or 10^9 times larger than the same ratio for ordinary lines.[2]

Only for the level 2^3 S of *He* may we expect such a long mean life–time that the Case II may occur. A large proportion of *He* atom will be then in the state 2^3 S and in favourable conditions a considerable optical depth of the nebula in the corresponding series may arise.

Application to the Wolf–Rayet statrs. To determine which of our two cases is realised in the gaseous shell surrounding a Wolf–Rayet star, the knowledge of W is required. We have no data about this subject but it seems that W will be scarcely smaller than 10^{-8}, or perhaps larger. We know, indeed, that during a month after the outburst, the Novae develop many features of the Wolf–Rayet Spectrum. Taking the velocity of the expansion of the gaseous shell 1000km/sec and the radius of the star after the ejection of gases 10^6 km, we obtain for W at the end of month the value $0.5 \cdot 10^{-7}$. For such value of W the formula (16) will be applicable to the levels with mean life–time longer than

[2]Observations have shown that the number of excited atoms in the ordinary excited states of hydrogen is of the order 10^4 per square centimetre of the nebular disc. The number of hydrogen atoms in the state 2 S will be therefore 10^{12} or 10^{13} per square centimetre and the optical thickness of the nebula in the first two lines of Balmer series may reach 0.1 or 1.

10^{-1} sec. Some of the metastabile states will have longer mean life–time. Such is undoubtedly the state 2^3 S of orthohelium. The accumulation of atoms in this state may cause considerable optical depth in the lines of the principal series of orthohelium. The number of atoms in the state 2^3 S of *He* per square centimetre of the surface of the envelope may be estimated in the following manner.

Not all quanta capable to ionize the normal *He*–atom emitted by the central star are absorbed by gaseous envelope, because in the opposite case at the temperatures of the Wolf–Rayet stars the lines of *He* would be much stronger than the lines of He^+. To explain the observed intensities of *He*–lines, let us suppose that about one per cent of the mentioned quanta are absorbed. The optical thickness of the gaseous envelope for the frequencies lying behind the frequency of ionization of normal *He* then will be about 0.01. If the absorption coefficient per each *He*–atom is of the same order as the absorption coefficient behind the head of the Lyman series of H, the number of normal *He*–atoms per square centimetre will be therefore about 2.10^{15}. Applying the formula (16) we find that the number of atoms in the state 2^3 S of *He* per each square centimetre of the surface of the envelope will be about 10^{14}. Such number of atoms will produce a considerable optical thickness of the envelope in the lines of the principal series of orthohelium.

The Collisional Excitation. Now we consider an atom which has only two levels 1 and 2 with the energies ε_1 and ε_2. We suppose that the collisions of the second kind may exite some nebular atoms to the metastabile state. The atom may after that pass into normal state either spontaneously emitting a quantum of the forbidden line or transmitting the energy of the excitation to a free electron. All other types of the transitions neglect. Let $b_{12}\,dt$ be the probability of an inelastic collision which excites the normal atom and a_{21} be the probability of the transition of an excited atom in the normal state by means of a superelastic collision. The condition of stationarity will have the form:

$$b_{12}\,n_1 - (A_{21} + a_{21})\;n_2 = 0. \qquad (19)$$

In the case where the velocity distribution of electrons obeys the Maxwell's law, we have

$$b_{12} = \frac{g_1}{g_2} \cdot a_{21} \exp\left(-\frac{k\nu_{12}}{k\,T}\right).$$

(19) becomes

$$\frac{n_2}{n_1} = \frac{g_2}{g_1}\frac{a_{21}}{A_{21} + a_{21}} \cdot \exp\left(-\frac{k\nu_{12}}{k\,T}\right).$$

If $A_{21} > a_{21}$, i.e. if the density of electrons is low, the ratio $\dfrac{n_2}{n_1}$ is smaller than

$$\frac{g_2}{g_1} \cdot \exp\left(-\frac{k\nu_{12}}{k\,T}\right).$$

In this case the spontaneous transitions are predominant and the forbidden lines appear in their full strength. If $A_{21} < a_{21}$, the forbidden lines will be weakened or will disapper altogether.

R E F E R E N C E S

1. Zeitschrift für Astrophysik, 6, 107, 1933.

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