## MULTIPLE SYSTEMS OF TRAPEZIUM TYPE

**Introductionary remarks.** After the instability of O–associations has been established in 1948, it was decided to pay attention to the pecularities of stars in these associations. In the beginning of 1949 B. Markarian and the author had noticed that in the central part of Cygnus association within a small cluster IC 4996 exists a multiple star ADS 13626 which differs from the majority of multiple stars. This reminded of the presence within Orion association of the Trapezium of Orion. An interesting photograph of that system can be found in the Burakan Atlas of Open Clusters. In the paper by B. Markarian and the author on the association around P Cygni [1] was introduced the concept of Trapezium type multiples. In 1950 and 1951 the important papers [2,3] of Markarian have been published in which the great role of Trapezium type systems within the O–clusters was disclosed. They were followed by two papers [4,5] by the author on Trapezium type systems. In 1952 P. P. Parenago [6] has published his report based on the treatment of all observations of the Trapezium of Orion where conclusions were rather in favour of positive energy of this system. In 1954 Sharpless [7] confirmed close inter connections between systems of Trapezium type and O–clusters. He also examined connections with diffuse nebulae and has found a number of new systems of Trapezium type.

At this stage further measurements of relative positions of components of these systems as well as determination of their spectral types are important.

The term "Trapezium" or "systems of Trapezium type" is now generally accepted, although among such systems we find triples, five member groups and even groups with more components. However the definition we give below seems quite clear in spite of some uncertainty of the boundary between the Trapezia and multiples of ordinary type. In this paper we try to give a rewiew of the present stake of study of such systems.

### §1. General considerations

The majority of multiple systems known to us have the following property: in them it is impossible to find three components $a, b, c$ such that the distances $ab$, $ac$ and $bc$ have the same order of magnitude. Multiple systems having such property we will call the "systems of ordinary type" or the ordinary systems.

A good example of an ordinary system is $\varepsilon$ Lyrae. It consist of two pairs. The distance between these pairs is about $208''$ while the distances between two components of $\varepsilon^1$ Lyrae is $3''.1$, and between the components of $\varepsilon^2$ Lyrae it is $2''.3$. Clearly, in any triple we can choose from this system, one of the distances will be by two orders of magnitude larger than the two remaining distances.

If in a multiple star it is possible to find three component for which the all three distances are of the same order of magnitude, then we call it a system of Trapezium type. Remarkably enough in $\theta'$ Orionis all the six mutual distances are of the same order of magnitude.

Evidently our class of multiples includes systems which are not even approximately resemble trapezia considered in geometry.

Of course for compiling a list of Trapezium type multiple stars it is neccessary to define exectly which distances are considered to be "the same order". Let us assume that two distances of the same order if their ratio is between 1/3 and 3.

It is well known that among the triple stars a great preponderance of the ordinary type triads is observed. However, the picture contains many interesting details.

Among the stars which are nearer to us than 10.5 parces there are seven triples. In all seven cases the principal component belongs to the main sequence. Table 1 contains the values of logarithms of halfed great axes of orbits of the furthest and the nearest satellites (expressed in AU) as well as the ratio $\kappa$ of great axes. In the last column the spectral types of components ( when they are known) are given.

| *Star* | $\lg a_1$ | $\lg a_2$ | $\kappa$ | *Spectra* |
|---|---|---|---|---|
| 40 Eri | 1.53 | 2.72 | 16 | K1, wA, M6 |
| $\alpha$ Cen | 1.37 | 4.12 | 563 | G0, K5, M5e |
| $-8^\circ$ 4352 | 0.11 | 2.80 | 490 | M2, M5 |
| 36 Oph | 1.50 | 3.72 | 166 | K2, K1, K6 |
| HR 6426 | 1.10 | 2.44 | 22 | K3, K4, M2 |
| $\mu$ Her | 1.07 | 2.59 | 33 | G7, M4 |
| $\begin{cases} -32^\circ & 16135 \\ -31^\circ & 17815 \end{cases}$ | 1.31 | 4.52 | 1600 | M5, M5, M1 |

**Table 1.**

From this table is seen that 1) the smallest value of $\kappa$ is 16 i.e. among the nearest stars there is no system of Trapezium type, and 2)the mean value of $\log \kappa$ is 2.27 i.e. the geometrical mean of all values of $\kappa$ is about 200.

Such large values of the ratio $\kappa$ allow to reduce, in the first approximation the motions in the triple system to the simple keplerian motions by elliptic orbits.

This conclusion for multiples situated in the solar neighborhood is confirmed by an example of the quadruple $\xi UMa$ which is also inside the same sphere of 10.5 parsec radius. It is a visual pair with the half-axis of 18 AU, of which both components are spectral doubles with great axes respectively, 1.5 and 0.04 AU. In this case too, the system is very far from being a Trapezium . This means that the motions again are reducible approximately to simple elliptic motions.

The picture changes only slightly when, instead of nearest stars we consider the stars which have high apparent brightness in Aitken catalog.

Of all stars brighter than 4.0 apparent magnitude and to the north of $\delta = -30^\circ$ only 15 have two or more physical visual components. They are distributed according their spectra in the following types:

| O – B2 | B3 – B9 | *A* | F – G | K – M |
|---|---|---|---|---|
| 4 | 3 | 3 | 3 | 2 |

From these 15 systems only 2 ($\zeta$ Persei and $\sigma$ Orionis) are systems of Trapezium type. Thus even in this case the ordinary systems are prevailing. It should be taken into account, that in the above 15 systems enter only multiples in which at least three components can be visually resolved. But to the north of $\delta = -30^\circ$ there are still more than 16 visual doubles brighter than $4^m.0$ in which at least one conponent is a spectral double. All these are multiples of ordinary type. Together we have 31 multiples brighter than $4^m.0$. They are distributed by the spectral classes as follows

| O – B2 | B3 – B9 | *A* | F – G | K – M |
|---|---|---|---|---|
| 8 | 3 | 5 | 8 | 4 |

Rougly speaking, this distribution by to spectral intervals is uniform. But it is significant that both Trapezium type systems just mentioned belong to the same interval O – B2. Is this fact and the fact of absence of stars of O – B2 type within the distance 10.5 parsec around the Sun interrelated ?

Let us take the stars brighter than $5^m.5$ to the north of $-30^\circ$. Now we have 18 stars which are main components of Trapezium type systems. They are distributed by spectral types in the following way

| O – B2 | B3 – B9 | *A* | F – G | K – M |
|---|---|---|---|---|
| 5 | 6 | 1 | 3 | 3 |

We again have a strong prevalence of O – B stars. This prevalence will become stronger if we exclude the cases where the satellite is very faint or very far from the brightest star, since in such cases the probability of optical satellite is rather high. To possibly avoid such cases let us introduce some limits for the distances from principal stars. For example for satellites of different visible magnitudes we can use the following distance bounds:

| m | d |
|---|---|
| 11.5 – 12.5 | $10''$ |
| 10.5 – 11.5 | 30 |
| 9.5 – 10.5 | 50 |
| 8.5 – 9.5 | 80 |

The cases where the components are fainter than $12^m.5$ are all excluded. Then only 11 stars will remain in our list, see Table 2. In the last column we give the apparent magnitudes of the components whose relative positions provide ground for describing the system as a Trapezium . A question arises: what are the probabilities for the objects in this list to be optical Trapezia. To give an answer to this question let us assume that the optical component is the faintest in the group (which is the most probable case).

Let us take into account that the mean stellar magnitude of the faintest components of our systems is $10^m.1$ and that the number of stars brighter than 10.1 in the equatorial galactic zone is about 10 per square degree. Transformation of a double star into a triple of Trapezium type occures if a background star projects within a circle around the double star whose radius is of the order of $50''$. An elementary calculation yields the probability of transformation of the optical binary into a Trapezium type configuration: it is of the order of $\frac{1}{180}$.

| *Star* | *ADS* | *HD* | *Spectra* | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|---|---|---|
| $\zeta$ Per | 2843 | 24398 | B1 Ib | 2.9 | 9.3 | 11.1 |
| $+14°796$ | 3579 | 31764 | B8 | 5.2 | 6.7 | 9.0 |
| 14 Aur | 3824 | 33959 | A2 | 5.2 | 7.2 | 11.0 |
| $\theta^1$ Ori | 4186 – 8 | 37022 | O7 | 5.4 | 6.8 | 6.8 |
| $\sigma$ Ori | 4241 | 37468 | O9.5V | 4.0 | 7.5 | 10.3 |
| 30 CMa | 5977 | 57061 | O9 III | 5.0 | 10.5 | 11.2 |
| P Pup | 6205 | 60863 | B8 | 5.2 | 9.3 | 10.0 |
| $\zeta$ Mon | 6617 | 67954 | G0 | 5.0 | 8.5 | 10.7 |
| $-21°4908$ | 11169 | 166937 | B8p | 4.0 | 9.5 | 9.5 |
| 59 Cyg | 14526 | 200120 | B3ne | 4.7 | 9.0 | 11.5 |
| $+34°4371$ | 14831 | 202904 | B3ne | 4.6 | 10.2 | 10.2 |

**Table 2.**

However a system is listed as a Trapezium , only if the mutual positions of its components satisfy certain additional conditions. Thus the optical (projected) component must not be too near to other two or too far from them. The probability that randomly projected star will satisfy these conditions must be about 1/2. Therefore the probability that any given double star will be transformed by means of projection into a configuration of three stars satisfying Trapezium conditions with values of parameters as in Table 2 will be less than $\frac{1}{2} \cdot \frac{1}{180} = \frac{1}{360}$.

To the north of $\delta = -30°$ there are 444 double stars with the principal star brighter than $5^m.5$. Therefore the expected number of Trapezium configuration formed owing to projection, with principal star brighter than $5^m.5$ will be 1.2. We can expect that the number of such optical systems in the Table 2 is 1 or 2.

The distribution of stars of Table 2 according to spectral classes is:

| O – B2 | B3 – B9 | *A* | F – G | K – M |
|---|---|---|---|---|
| 4 | 5 | 1 | 1 | 0 |

Thus from 11 systems of Trapezium type 9 have O –B stars as the main component, i.e. the early spectral types are strongly prevalent. Possibly, the two systems of spectral interval A – G are optical triples. It is also possible that they are in fact ordinary systems which acquired the false appearance of a Trapezium via projection.

From the systems in Table 2, all four that belong to O–B2 spectral interval are members of some O–association. In particular, $\zeta$ Persei is a member of Perseus II, the stars $\theta'$ and $\sigma$ Orionis are members of the association in Orion while the system 30 Canis Majoris belongs to the cluster NGC 2362 which is a nucleus of a group of hot giants.

From stars of the later spectral classes, the star ADS 11169 ($\mu$ Sagittarius, according to more refined classification it is of type cB8e) enters apparently in the association Sagittarius I. Other stars of B3 – B9 type apparently are not members of O–associations.

We conclude about close interconnection between the Trapezium type systems, the principal stars of which belong to the spectral interval O – B2 and the O–associations.

## §2. Instability of Trapezium type systems

The fact that the overwhelming majority of multiple systems are of ordinary ( and not of Trapezium) type has for a long time attracted atention of the astronomers. In such systems the motions are approximately of Kepler type. Evidently such motions can be maintaned for a very long time and these systems are stable. In this respect the multiples of ordinary type make a sharp contrast with the open clusters where owing to the exchange of kinetic energies between members of the cluster some members may acquire energies sufficient to escape. This leads to disintegration of clusters. In the multiples of ordinary type such energy exchanges are rather rare and the system remains stable for very long time.

The situation is quite different in the Trapezium type systems. Here the motions are similar to those within clusters. Therefore there are real chances for the members to escape. But owing to the small number of members the lifetime here must be shorter. The small dimensions of these systems also imply a shorter lifetime.

The formula for the disintegration time for a cluster, when applied to Trapezium type systems leads to the timescales of the order of $2 \cdot 10^6$ years. In many cases this means that a Trapezium type system can disintegrate during a period in which every star makes only a few crossing of the system.

This conclusion is physically clear. Even for a small number of crossings, a component has considerable chances to approach one of the remaining components at a distance where the energy of their interaction may exceed the value of the escape energy. The chances of expulsion of a star from a system are cosiderable.

Arguments of this type lead to the conclusion that lifetimes of Trapezium type systems are of the order of one or two millions of years. Thus their components are exstremely young stars.

We do not exclude the theoretical possibility of some periodic or quasi–periodic motions in the Trapezium systems. But such motions require very special initial conditions and therefore have exceedingly small probability.

## §3. On the sign of the energy of Trapezium type systems

We know that in the Galaxy alone there are billions of double and multiple stars of which the prevailing majority are stable systems. This means that the values of energy in prevailing majority of cases should be negative. Although this conclusion is based on purely statistical considerations, it is confirmed by direct determination of orbits for many of them. Nevertheless, it is possible that some pairs or multiples (minority of them) have positive energies. Some of the Trapezium type systems can be considered as possible carriers of positive energy. This would mean that they are recently formed groups and now are in the process of disintegration.

It is interesting that in stellar systems which are related to Trapezium type multiples such as O–associations and O–clusters, the sign of the total energy is at least sometime positive.

If among Trapezia there are groups of positive energy, it is easy to calculate the time they needed to reach the present sizes. In this way we find out that some Trapezium systems have the age of the order of $10^5$ years and in any case less than $10^6$ years.

According to Parenago, our conclusion on the positive sign of total energy at least of some Trapezium is confirmed by data on movements in the Trapezium of Orion itself ($\theta'$ of Orion).

For the majority of Trapezia, the time during which they were observed is not sufficient to decide conclusively about the sign of their total energy. Unfortunately, all such systems, being comparatively wide groups, have not received sufficient attention of observers.

## §4. Some very wide systems

On some photographs of regions around O–associations we find wide groups which are the result of expansion of Trapezium type groups. Let us take for example the region in Cygnus around NGC 6871. On the photographs of that region there are at least five groups which are of this type, even after when we exclude the similar groups in NGC 6871 itself. The spectral types and stellar magnitudes of stars in this region were determined by Enier [10]. In all these systems the brihtest star belongs to the B type. The catalog numbers and coordinates of these stars due to Enier are given in the Table 3. It turns out that all are of B type. In the fourth column the numbers of components ( multiplisity) is given, in seventh the largest distance between the components expressed in seconds of arc. In the next column the same distance is expressed in AU, the distance of the system from us is accepted to be 1500 parsec. Geometrically these systems are similar (especially the systems A 34°140 and A 35°190) to Trapezium in Orion. However in linear units they are about ten tines wider.

| *Star* | $m_{pg}$ | $S_p$ | $n$ | $\alpha\,(1950)$ | $\delta\,(1950)$ | $d_{max}$ | $D_{max}$ | *Note* |
|---|---|---|---|---|---|---|---|---|
| A 34°140 | 10.00 | B5 | 4 | $20^h03^m.8$ | $31°10'$ | 52 | 78.000 | |
| A35°190 | 9.56 | B2 | 4 | 05.0 | 35 09 | 93 | 140.000 | |
| A 35°240 | 8.22 | B2 | 5 | 07.1 | 35 22 | 55 | 83.000 | BD + 35°3987 |
| A 35°254 | | B2 | 4 | 07.4 | 35 20 | 41 | 61.000 | |
| A 35°283 | 8.67 | B3 | 6 | 08.0 | 35 43 | 78 | 117.000 | BD + 35°4004 |

**Table 3.**

The fact that the brightest stars of these groups belong not to the classes O or BO, but to the later subtypes of B speaks in favour of their older age as compared with the stars of our List of Trapezium system. This is especially true as regards A 34°140 and A 35°190 which are resemble Trapezium of Orion itself but are wider by one order of magnitude.

An expansion which is a result of interactions of stars ( like in open clusters) is necessarily accomponied by a change of the shape of the group. Since here the shape remains similar to that of Trapezium of Orion itself, this speaks rather about the expansion resulting from the positive total energy of the group.

## §5. The narrow systems of Trapezium type

To understand the evolutionary role of the Trapezium systems it is important to study most narrow cases. Special attention must be paid to the visually single O or BO stars which sometimes consist of several components. This question was treated in the paper of Sharpless [7]. It seems that such narrow system can persist not more than a few hundreds years. Therefore the probability to find such systems among visually single stars must be very low. Nevertheless, it is worthwhile to prove the absence of such single stars. At the same time it is necessary to try to find narrow visual multiples of Trapezium type. If the distances of such multiples are of the order of a thousand parsecs, this would mean that we try to find groups having sizes less than $4''$. ADS 719, ADS 6033, ADS 11344, ADS 364, ADS 4164 and ADS 14010 are such systems. The brightest star of the first of these systems is of type O6. The brightest star of the second system is known as variable VY CMa and belongs to the class Ma. The largest distance between its components is equal $2''.9$. Since the variable giants of late type are in a way related to the early type stars, it is possible that here we deal with a real Trapezium. Thus it is possible that we have here an additional evidence of close relation between blue and red giants. Unfortunately, data on components of other systems of these group are absent. We would like to stress the importance of spectral studies of components of these interesting multiplies. In any case the scarcity of these systems is in complete accordance with the idea about their instability.

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Byurakan
Astronomical Observatory of the Academy
of Sciences of Armenia.