TO THE STATISTICS OF DOUBLE STARS

It was indicated by a number of authors, that the study of distribution law of elements of double stars orbits, as well as of other statistical interrelations for these objects, can give interesting results for cosmogony in general and for the age problem of our star system in particular. However, as indicated by the author in the preliminary note [1], wrong conclusions are often made from observational data. The aim of this study is to show the erroneous nature of some old conclusions widely spread in the literature [2]. We also indicate some new implications from the observational material concerning double stars.

Contrary to Jeans the observed distribution of eccentricities among the double stars with known orbits is far from giving a proof of equipartition of energies. A direct consideration of energies of double stars (or large semi-axes of their orbits) proves, that the equipartition of energy does not occur even among wide pairs. This circumstance, together with the absence of dissociative equilibrium between double and single stars, leads to an upper age bound of 10^{10} years for the ensemble of double stars.

1. Distribution of eccentricities of the double star orbits

The distribution of eccentricities of double star orbits was discussed often enough. Namely it was established, that among the double stars whose orbits have been determined, the number of pairs with eccentricities less than ε is proportional to ε^2.
On the other hand Jeans has shown that under statistical equilibrium (Boltzman distribution) the same dependence should be observed. From this conclusions are made, that we already deal with the most probable distribution , and further about the long time scale. According to Jeans more carefull formulation, given in his response [3] to authors preliminary note, equipartition exists at least for some parameters.

First of all one should realize that the distribution of eccentricities now at hand can differ to a large degree from the real one on account of selectivity of observational material.

For the time being we know only the orbits of pairs with comparatively short periods. On the other hand, as observations show, the average eccentricity certainly increases with period. Therefore the true relative number of pairs with large eccentricities excedes the relative number of such pairs among the double stars whose orbits have been determined.

To make things clear from theoretical point of view, we consider the distribution of states of a satellite in the phase space.

For coordinates in the phase space we take three components of satellite position and three components of its impulse with respect to the main star. Under statistical equilibrium , the number of satellites dN in the volume element $dx\,dy\,dz\,dp_x\,dp_y\,dp_z$ of the phase space is:

$$dN = C \exp\left(-\frac{E(x,y,z,p_x,p_y,p_z)}{\theta}\right) dx\, dy\, dz\, dp_x\, dp_y\, dp_z, \tag{1}$$

where

$$E = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) - \frac{\gamma M m}{\sqrt{x^2+y^2+z^2}} \tag{2}$$

is the energy of satellite, M and m are masses of the main star and the satellite, γ is the gravitational constant and θ is the Boltzman distribution module.

We will consider a broader class of distributions. Suppose that in the phase space the density is an arbitrary function $f(E)$ of the energy E, rather than has a special form $C \exp -\frac{E}{\theta}$.
Then

$$dN = f(E)\, dx\, dy\, dz\, dp_x\, dp_y\, dp_z.$$

Now let us make canonical transformations in the phase space going over from the variables x, y, z, p_x, p_y, p_z to the variables L, G, H, l, g, and h of the Delonais lunar theory [4]. The first three of these quantities are in the following way expressed in terms of usual elements of the elliptic motion, which are the large semi-axis a, the inclination i and the eccentricity ε:

$$\begin{aligned} L &= m\sqrt{\gamma M}\, a^{1/2}, \\ G &= m\sqrt{\gamma M}\, a^{1/2}(1-\varepsilon^2)^{1/2}, \\ H &= \sqrt{\gamma M}\, a^{1/2}(1-\varepsilon^2)^{1/2}\cos i. \end{aligned}$$

The angular coordinates l, g and h represent the average anomaly, the distance of periastron from the node and the longitude of ascending node respectively.

It is known that under the canonical transformation of the phase space volumes remain intact (the Jacobian is equal 1). In other words

$$dx\, dy\, dz\, dp_x\, dp_y\, dp_z = dL\, dG\, dH\, dl\, dg\, dh\,.$$

On the other hand

$$E = -\frac{\gamma^2 M^2 m^3}{2L^2} = -\frac{1}{2}\frac{\gamma M m}{a}.$$

i.e. the energy depends solely on L. Hence, in our case the density in the phase space also depends on L only, and we can write:

$$dN = f(L)\, dL\, dG\, dH\, dl\, dg\, dh\,.$$

This implies that the number of pairs with L between L and $L + dL$, and $G \geq G_0$ is equal to

$$8\pi^3 f(L)\, dL \int_{G_0}^{L} dG \int_0^G dH = 4\pi^3 f(L)(L^2 - G_0^2)\, dL,$$

since l, g and h independently vary within $(0, 2\pi)$.

We have

$$L^2 - G_0^2 = m^2\gamma^2 M^2 a\varepsilon_0^2 = L^2\varepsilon_0^2,$$

where ε_0 is the eccentricity, corresponding to an orbit L, G_0.

Thus the number of stars with $\varepsilon < \varepsilon_0$ (i.e. $G > G_0$) and L between L and $L + dL$ is

$$4\pi^3 f(L) L^2 \varepsilon_0^2 \, dL,$$

which implies that the number of orbits for which $\varepsilon < \varepsilon_0$ is

$$N(\varepsilon_0) = 4\pi^3\varepsilon_0^2 \int_0^\infty f(L)L^2 \, dL. \tag{3}$$

We have the following theorem:

If the density in the phase space is a function of L, i.e. of total energy and solely of this quantity, then for arbitrary density function $f(L)$ the number of stars, with eccentricities less than ε_0, is proportional to ε_0^2.

It follows, that even if we assume that the observed $N(\varepsilon_0)$ is proportional to ε_0^2 (we have our doubts about this because of selectivity of the material) this does not imply, that the phase density is proportional to $\exp(-E/\theta)$, or in other words, that equipartition of energy holds. Actually for any distribution of energy under single condition that the phase density does not depend on other elements, we have $N(\varepsilon) \sim \varepsilon_0^2$.

Thus, even if we assume that in fact $N(\varepsilon) \sim \varepsilon_0^2$, we can not conclude about equipartition of energy, let alone the life period of a star system.

However we note the following circumstance. According to the above , in the case where the phase density depends on L only (equivalently on E or on the large semi-axes), for each interval dL we have, that the number of orbits with eccentricities less than ε is also proportional to ε_0^2. In other words, the number of orbits with eccentricities between ε and $\varepsilon + d\varepsilon$, should be proportional to $\varepsilon \, d\varepsilon$ regardless of a. Therefore the average eccentricity for each interval of values of large semi-axes equals

$$\overline{\varepsilon} = \frac{\int_0^1 \varepsilon^2 \, d\varepsilon}{\int_0^1 \varepsilon \, d\varepsilon} = \frac{2}{3} \tag{4}$$

independent of a.

The observational material is in countradiction with this, as demonstrated by the following table, obtained by Aitken [5]

$\overline{P}$	$\overline{\varepsilon}$	n
16.8 years	0.43	14
37.1	0.40	24
73.0	0.53	24
138	0.57	23
200	0.62	18

Table 1.

The table contains average eccentricities for stars grouped according to periods.

The first column gives the average period for each group while the last — the number of stars in a group. If we add to this the statistical result by Russel, which says that for stars with periods about 5000 years the average eccentricity is 0.76, we should conclude, that ε depends on P. It is known that $P \sim L^3$. Therefore $\overline{\varepsilon}$ depends on L. It becomes evident that the main assumption made above should be false and the phase density does not depend on large semi-axes alone. This means that the phase density is by no means proportional to $\exp(-E/\theta)$. Even the assumption that it depends on energy alone is false. There are indications that the given dependence of $\overline{\varepsilon}$ on P is subject to strong observational selection [10, 11, 12].

Possibly for distant components ($P > 100$ years) the variation of $\overline{\varepsilon}$ is small and phase density depends only on E. It could be interesting to study the dependence of phase density on E basing upon observations and to measure its deviation from Boltzman dependence.

Derivation of the phase density from observational data

In this paragraph we assume that the phase density depends solely on E. We will try to get the form of this dependence from the empirical material. We saw that at least for smaller values of L the phase density probably depends on other elements too. Hence our result should be treated rather as an average with respect to other elements. Even in this form our conclusion has some value, all the more, for distant components our assumption is probably valid. Let the phase density again be $f(L)$. This means that in the volume element $dx\,dy\,dz\,dp_x\,dp_y\,dp_z$ the number of stars is

$$f\left(\gamma M m \cdot \sqrt{\frac{m}{\frac{2Mm\gamma}{r} - \frac{p^2}{m}}}\right) dx\,dy\,dz\,dp_x\,dp_y\,dp_z,$$

where

$$r = \sqrt{x^2 + y^2 + z^2}, \quad p = \sqrt{p_x^2 + p_y^2 + p_z^2}.$$

Therefore the distribution density in the space is

$$\rho = \iiint f\left(\gamma Mm \cdot \sqrt{\frac{m}{\frac{2Mm\gamma}{r} - \frac{p^2}{m}}}\right) dp_x\, dp_y\, dp_z =$$

$$= 4\pi \int\limits_0^{m\sqrt{\frac{2M\gamma}{r}}} f\left(\gamma Mm \cdot \sqrt{\frac{m}{\frac{2Mm\gamma}{r} - \frac{p^2}{m}}}\right) p^2\, dp.$$

By the spatial distribution of satellites we mean the distribution obtained after parallel shifts of the pairs in a way bringing the main stars to a single point. The upper bound in the last integral is obtained from the condition, that our satellites, move along eliptical orbits, i.e. our system have negative total energy.

Now a change of variable

$$L = \gamma Mm \cdot \sqrt{\frac{m}{\frac{2Mm\gamma}{r} - \frac{p^2}{m}}}$$

in the last integral yields

$$\rho(r) = \int\limits_{m\sqrt{\frac{\gamma M}{2}}\, r^{1/2}}^{\infty} f(L)\sqrt{\frac{2Mm^2\gamma}{r} - \frac{\gamma^2 M^2 m^4}{L^2}} \cdot \frac{4\pi^2\gamma^2 M^2 m^4\, dL}{L^3}.$$

Denote

$$K = m\sqrt{\frac{\gamma M}{2}}\, r^{1/2},$$

then

$$\rho(r) = 4\pi^2\gamma^3 M^3 m^6 \int\limits_K^{\infty} f(L)\sqrt{\frac{1}{K^2} - \frac{1}{L^2}}\, \frac{dL}{L^3},$$

or

$$\rho(K) = C\int\limits_K^{\infty} f(L)\sqrt{L^2 - K^2}\, \frac{dL}{KL^4}. \tag{5}$$

Using this integral equation we can find the phase density $f(L)$ in terms of given ρ. Epic in [6] has shown that the observational material at hand, after correction for observational selectivity gives

$$\rho \sim \frac{1}{r^3}. \tag{6}$$

or

$$\rho \sim \frac{1}{K^6}.$$

It is evident that if ρ has this special form, then the function

$$f(L) \sim \frac{1}{L^3}. \tag{7}$$

satisfies the equation (5).

Let us compare this "observed" density in the phase space with that under statistical equilibrium where we have

$$f(L) = C\ \exp\left(-\frac{E}{\theta}\right) = C\ \exp\frac{\gamma^2 M^2 m^3}{rL^2\theta}. \tag{8}$$

It remains to determine the value of θ. If the double stars ensemble reached statistical equiblrium as a result of interaction with other stars, then θ by order should equal two thirds of average kinetic energy of translational movement of surrounding stars. Suppose that the average speed of translational movement of stars is 25 km/sec by order, then already for $a > 20\,A.U.$ the exponent on the right-hand side of (8) becomes much less then 1. Therefore for large values of L (as well as of a)

$$f(L) = \text{const} \tag{9}$$

is a satisfactory approximation.

Meanwhile the result obtained by Epic refers just to distant components. Hence (7) refers to larger values of L.

We see that the "observed" phase density (7) is quite different from that of statistical equilibrium (9).

One can show that "the distribution in the space" in the case of statistical equilibrium also differs strongly from one observed. Indeed from (9) and (5) follows that under statistical equilibrium

$$\rho \sim \frac{1}{r^{3/2}} \tag{10}$$

in contradiction with the observed distibution (6). The difference between (10) and (6) is so great that leaves no doubt that in fact (10) is unsatisfactory even as a rough approximation. The formula (3) was established by Epic for distant components ,up to 10.000 A.U.. We conclude that even for such distant components the encounters has not yet led to statistical equilibrium (that is to the most probable distribution) in the sizes of great semi-axes,i.e the energies. We will see that this strongly reduces the upper bound for the life period of a star system.

3. Testing Epic law of inverse cubes by new observational material

In the present paragraph we consider a very simple testing method for the law (6), obtained by Epic for the distribution of components in the space. We will see that this new method of analysis confirms an approximate correctness of formula (6).

As a matter of fact, if components are distributed around central stars by the law $1/r^n$, where n is arbitrary, then the projection of the distribution density onto the celestial sphere will be $1/r^{n-1}$.

Assume we have double stars aggregate governed by this distribution within a volume. For any shifts of the volume, the distribution of apparent distances obviously will continue to satisfy the law $1/r^{n-1}$. A summation of this distributions for volume elements both along a direction and in various directions also leads to $1/r^{n-1}$. Therefore for arbitrary large part of the sky, for apparent (projected) distances we obtain the same law.

In particular under the alternative assumptions

$$\rho \sim \frac{1}{r^3} \quad \text{and} \quad \rho \sim \frac{1}{r^{3/2}}$$

for distribution densities in projections we obtain

$$\rho \sim \frac{1}{r^2} \quad \text{and} \quad \rho \sim \frac{1}{r^{1/2}}$$

which implies that the number of stars with apparent distances between r_2 and r_1 is proportional to

$$\ln \frac{r_2}{r_1} \quad \text{and} \quad r_2^{3/2} - r_1^{3/2}. \tag{11}$$

From Aitken's catalogue [7] we took all stars with apparent magnitude up to 9.0 lying in the northern hemisphere (4640 stars altogether). In this bounds Aitken's catalogue seems to be sufficiently homogeneous, since all stars with magnitude up to 9.0 were tested by Aitken at Lick. The following table gives the number of pairs with distances ranging from $0.5''$ to $8''$. The second and the third lines give numbers proportional to $\ln \frac{r_2}{r_1}$ and to $r_2^{3/2} - r_1^{3/2}$ respectively. The proportionality coefficient C was chosen to have the same total number in each line.

Interval	$0.5 - 1''$	$1 - 2''$	$2 - 4''$	$4 - 8''$	*Total*
The observed number of pairs	883	1160	1283	1314	4640
$C \ln \frac{r_2}{r_1}$	1160	1160	1160	1160	4640
$C \left(r_2^{3/2} - r_1^{3/2}\right)$	136	382	1080	3040	4638

It is seen from this comparison that $C \ln \frac{r_2}{r_1}$ indeed yields an approximation (with about 10% precision) to the observed numbers, while $C \left(r_2^{3/2} - r_1^{3/2}\right)$ can not be justified.

The deviations from $C \ln \frac{r_2}{r_1}$ will definitely become smaller if optical pairs are excluded.

Thus Epic's law $\rho \sim \frac{1}{r^3}$ is confirmed in the first approximation. Once again this shows that energies of stellar pairs are not distributed by Boltzman law.

In his answer to the author's preliminary note Jeans acknowledged [3], that equipartition in energies does not exist, but he added that "in certain respects there is a tolerably good approximation to equipartition ". However, in view of the above corollaries from Epic's law there can be no word on any approximation to equipartition at all.

Relaxation time for a double stars ensemble

Let us consider the time required for a double stars ensemble in a star system to reach the statistical equilibrium with surrounding stars. In statistical equilibrium we usually have two processes acting in opposite directions: on one hand, the destruction of physical pairs in cosequence of interaction with stars of the field and, on the other hand, formation of pairs when three initially independent stars come together. In the latter case the third body caries away the excess energy. Below we show that, because of absence of statistical equilibrium in our star system, complete mutual compensation of these process does not take place: the number of pairs formed is negligibly small in comparison with the number of pairs destroyed.

Along with the destruction of pairs by approaching stars, small energy variations may accumulate to cause a destruction. These processes lead to a statistical equilibrium in the sence of Boltzman distribution .

It will be evident that the average time of destruction of a star pair, which we calculate below, is quite enough to reach Boltzman distribution .

Boltzman distribution is reached by means of energy variations smaller than those needed for destruction. Therefore the time required for this is not greater than the average lifetime of a pair. Thus the average life necessary for destruction of a pair gives the order of "relaxation time" of a double star system. Our computations will refer to "distant pairs" with a distance between components larger than 100 A. U. and thousands A. U. in average by order.

A passage of the third star near a stellar pair can be of two types:

1) the minimal distance of the passing body from the center of gravity of the pair is large as compared with the great semi-axes of the orbit;

2) the minimal distance of the third body from one of components is small in comparison with the large semi-axes of the pair.

The corresponding types we call "distant" and "close" passages. Passages of intermediate type also may occur, but we will not dwell on them, since their role is secondary.

Considering passages of Coulomb particles near an atom, Bohr has shown [8,9] that the role of distant passages is negligably small as compared with the role of close passages. Therefore we will consider close passages only. The contribution of distant passages somewhat shortens the relaxation time but leaves its order intact.

For pairs of the type we now consider, velocities of orbital motions around the gravity center has the order of one or at most 2 or 3 km/sec. Meanwhile the relative velocities in the star system are about 30 km/sec. Therefore in the coordinate system attached to the gravity center, the satellite can practically be considered to be motionless.

Due to the mentioned velocity ratio a closely passing star will deliver most of its influence on the satellite during a small fraction of the rotation period of a pair.

The satellite will acquire extra kinetic energy while its potential energy will remain unchanged. As a result, we will have either growth of greater semiaxis or complete destruction of the pair. A contrary pattern of interaction requires smaller kinetic energy of the passing star as compared with that of the satellite. However, the probability of such an event is too small.

The above remarks imply that to influence the satellite, the central star needs a time period much greater than the duration of an encounter.

In this way we come to the problem of evaluation of the change of kinetic energy of a satellite under influence of a passing star, in the coordinate system attached to pair's center of masses.

A simple calculation gives the energy increase due to a distant passage to be

$$\Delta E = \frac{mv^2}{2}\,\frac{1}{1+\frac{p^2v^4}{4m^2\gamma^2}}, \tag{12}$$

assuming the masses of the passing star and the satellite are equal. Here p is the encounter parameter, i.e. the distance from the satellite from the line along which the passing star moved before encounter. The number of encounters for which p lies between p and $p+dp$, the velocity of the passing star between v and $v+dv$, and which occur within time interval dt equals

$$2\pi p\,dp\,v\,dt\,dn,$$

where dn is the number of stars within unit volume possesing velocities between v and $v+dv$. Therefore the energy increase during time t will be

$$\pi t\int mv^3\,dn\int\frac{p\,dp}{1+\dfrac{p^2v^4}{4m^2\gamma^2}};$$

Integration in p is over values corresponding to a close encounter, i.e. over $p<a$, a is the great semiaxis.

Therefore

$$\Delta E = 2\pi t m^3\gamma^2\int\frac{\lg\left(1+\dfrac{a^2v^4}{4m^2\gamma^2}\right)}{v}\,dn,$$

or

$$\Delta E = 2\pi t m^3\gamma^2\frac{n}{\overline{v}}\lg\left(1+\frac{a^2\overline{v}^4}{4m^2\gamma^2}\right), \tag{13}$$

where n is the total number of stars in unit volume (the stellar density), $\overline{v}$ is the mean velocity. The time required for ΔE to reach the total energy of the system which is $-\dfrac{\gamma m^2}{2a}$ is

$$t = \frac{\overline{v}}{4\pi m\gamma an \lg\left(1+\frac{a^2\overline{v}^4}{4m^2\gamma^2}\right)}. \tag{14}$$

This we can (and do) consider to be the relaxation time. Here a is some mean value over this period which is close to the initial value a_0, since for the main portion of interaction time the values of a remain less than a_0.

Let us substitute in (14) $\overline{v} = 3\cdot 10^6$ cm/sec and $m =$ the mass of Sun. The observed values of A can reach $1/20$ parsec, while $n = 0.1(\text{parsec})^{-3}$. We get the value $t = 5\cdot 10^9$ years. For smaller values of a we get values of order 10^{10} and 10^{11} years.

Thus, for double stars with distances between components less than 10000 A. U., Boltzman distribution is reached only after a period of 10^{10} years. The Epic distribution $\left(\rho \sim \frac{1}{r^3}\right)$ was derived for pairs from just this class. Hence for them Boltzman distribution is not valid. We conclude that the age of these pairs cannot exceed 10^{10} years. In other words, the distribution of great axes of double stars orbits favours rather definitely the "shorter time scale".

Our previous note mentioned this circumstance, albeit above calculation of the relaxation time was not presented there. This gave Jeans a chance to write: "I cannot see that Prof. Ambartzumian's remarks in any way challenge this position, so that, it seems to me the observational data he mentions are not opposed to the long time scale of 10^{13} years, but only to an infinitely long time scale".

Meanwhile we have seen, that simple calculations indicate that observational data in question are in contradiction not only with the time scale of 10^{13} years, but even with the time scale of 10^{11} years, i.e. they favour completely the shorter time scale.

Dissociative equilibrium for double stars

Another important data confirming that the encounters have not by now created a statistical equilibrium for pairs with distances about 10^4 A. U. by order, are deviations in the number of such pairs from what is expected under dissociative equilibrium .

We denote by δn_D the number of pairs whose satellites lie within element $\delta\Gamma$ of the phase space which we considered above. According to standard rules of kinetic theory of gases, by a dissociative equilibrium we have

$$\frac{\delta n_D}{n^2} = \frac{\delta\Gamma}{(\pi m\theta)^{3/2}} \exp\left(-\frac{E}{\theta}\right), \tag{15}$$

Here E is again the internal energy of a pair with satellite in $\delta\Gamma$, θ is the module of the Boltzman distribution for motion of the stars, n is the number of single stars in unit volume. If we choose $\delta\Gamma$ in that part of the phase space where $a > 100 A.U.$, then the factor $\exp -E/\theta$ can be replaced by 1. We will have

$$\frac{\delta n_D}{n^2} = \frac{\delta\Gamma}{(\pi m\theta)^{3/2}}, \tag{16}$$

By summation this formula extends to volumes $\delta\Gamma$ which are no longer infinitesimal. The only restriction is that $\delta\Gamma$ should not include parts where E/θ is no longer small as compared with 1.

We can (and do) take that part of the phase space where $a_1 < a < a_2$ for some bounds a_1, a_2. The corresponding bounds for L will be

$$L_1 = m\sqrt{\gamma M}\, a^{1/2} a_1^{1/2} \quad \text{and} \quad L_2 = m\sqrt{\gamma M}\, a^{1/2} a_2^{1/2}.$$

The corresponding phase volume is found to be

$$\begin{aligned} \delta\Gamma = 8\pi^3 \int\limits_{L_1}^{L_2} M \int_0^L dG \int_0^G dH = \frac{4\pi^3}{3}\left(L_2^3 - L_1^3\right) = \\ = \frac{4\pi^3}{3} m^3 \left(\gamma M\right)^{3/2} \left(a_2^{3/2} - a_1^{3/2}\right). \end{aligned} \tag{17}$$

Substituting (17) into (16) and putting there $M = m$ we obtain

$$\frac{\delta n_D}{n^2} = \frac{4}{3} m^3 \left(\frac{\pi\gamma}{\theta}\right)^{3/2} \left(a_2^{3/2} - a_1^{3/2}\right). \tag{18}$$

For $a_1 = 10^2$ A. U., $a_2 = 10^4$ A. U. and for the same numerical values of contstants as used above, we get

$$\frac{\delta n_D}{n} = 10^{-8}.$$

This is the fraction of double stars which under dissociative equilibrium will have a satellite with a between 100 and 10^4 A. U.. In reality, at least one from every several dozens has this property, i.e. the event in question occures millions time more often then it should under dissociative equilibrium .

This perhaps is the most striking evidence, indicating that our Galaxy is very far from the statistical equilibrium state and, in conjunction with the results of the previous section, speaks for the validity of the shorter time scale of 10^{10} years.

Because at present we have in our Galaxy an excess number of double stars as compared with the equilibrium state, dissolutions occur considerably (perhaps million times) more often then creations of pairs.

The result of this section we roughly formulate as follows.

The existence of such pairs as α and Proxima Centauri or Washington 5583–5584 proves the validity of the shorter time scale. In fact, satellite–stars having a about 10^4 A. U. are so numerous that even the star closest to us possesses such a satellite.

Conclusion

Until now it was a widespread opinion, mainly due to Jeans, that statistic of double stars speaks in favour of the longer time scale. While new facts from other domains of astronomy kept confirming the shorter scale, double stars remained the main argument for longer evolutionary scale. In the present paper the latter argument is shown to be an illusion. A proper treatment of statistical data very definitely points to the shorter time scale.

R E F E R E N C E S

1. Nature, vol. 137, 537, 1936.
2. Nature, vol. 136, 432, 1935.
3. Nature, vol. 137, 537, 1936.
4. Frank und Mises, " Die Differential und Integralgleichungen der Mechanik und Physik," Zweiter Teil, SS, Braunschweig, pp. 158–162, 1927.
5. Aitken, The Binary Stars, 1935.
6. Tartu Observatory Publications, vol. 25, 1924.
7. Aitken, The New General Catalogue of Double Stars, Washington, 1933.
8. Bohr, Phil. Mag., vol. 25, 10, 1913.
9. Bohr, Phil. Mag., vol. 30, 581, 1915.
10. H. B. Seyfert, 896, 1934.
11. C. R. Barbier, vol. 199, 930, 1934.
12. M. N. Finsen, vol. 96, 862, 1936.

4 December 1936

Astronomical Observatory
of Leningrad University.