ON THE SURFACE BRIGHTNESSES IN GALAXY

§1. **Integral surface brightnesses .** Stellar counts are one of the main tools in any study of the structure of our Galaxy. However the use of stellar counts is connected with certain difficulties both of principal and practical nature. In particular this demands knowledge of the luminesity function and its changes in space.

However some data on the structure of Galaxy is possible to obtain avoiding detailed analysis of stellar counts. Sometimes it is sufficient to study the surface brightnesses produced by stars, i.e. the sums of apparent brightnesses of stars, seen in a unit solid angle (for example per square degree). We shall use below the concept of "integral surface brightness" in this particular sense. This means that other sources of brightness, such as diffuse illumination of the night sky or the zodiacal light are excluded. We exclude also the "galactic light" i.e. the light scattered by interstellar dust and by diffuse nebulae, as well as all extragalactic sources.

Unfortunately at the moment the only source of data concerning total surface brightnesses is the integration of counts of stars of different magnitudes according to formula

$$I = \int_{-\infty}^{+\infty} A(m) 10^{0.4(m_0 - m)}\, dm, \tag{1}$$

where the unit in which I is measured is one star of magnitude m_0 and $A(m)$ is the number of stars per square degree with magnitudes between $m - \frac{1}{2}$ and $m + \frac{1}{2}$. Of course the quantity I is a function of celestial coordinates.

An attempt to determine the quantity I from observed brightness of the sky was made in a recent paper of Fesenkov [1]. Further accumulation and improvement of observations of this type is extremely desirable.

Calculating I following (1), we meet difficulties of two types. Sometimes faint stars (weaker than 20.0) which avoided the count make a substantial contribution to the integral (1). This contribution should be calculated by means of extrapolation. On the other hand, for brightest stars the values of $A(m)$ become indefinite. Owing to this, I is strongly dependent on occasional presence of one or two brighter stars in the field under consideration.

Nevertheless the counts of Van–Rhijn together with the data on individual brightest stars enable to determine approximate values of I for different directions in the sky.

§2. **Partial surface brightnesses .** Along with the total surface brightnesses we can consider the partial surface brightnesses caused by the radiation of stars of certain physical type. Thus, we can speak about the partial surface brightness as generated by F type or G type stars.

The partial surface brightnesses are calculable by means of the same formula (1), where $A(m)$ now represents the numbers of stars of the given type.

The mentioned difficulties of calculation of total surface brightnesses are present also for the partial surface brightnesses , especialy owing to scarcity of data on spectral types of faint stars. The material for calculation of partial surface brightnesses for different spectral classes is supplied by the spectral catalogues like HDC, BSD and HDE. Of course if the lists of stars of different types are not overlapping, then the sum of the partial surface brightnesses will be equal to the total surface brightness.

Some astronomers have recently introduced the concept of a subsystem of stars of same physical type (Lindblad [3], Kukarkin [4]). Evidently the total surface brightness is the sum of surface brightnesses calculated for such subsystems.

§3. **The emission coefficients.** Calculating the surface brightnesses we have to consider the total emission coefficient as well as the partial emission coefficients. The total emission coefficient η is defined to be the total brightness of all stars in a cubic parsec expressed in units equal to the radiation of a star of some standard absolute magnitude M_0.

If $\Phi(M)$ is the luminasity function normalized in such a way that

$$\int \Phi(M)\, dM = n, \tag{2}$$

where n is the total number of stars in a cubic parsec then

$$\eta = \int \Phi(M) 10^{0.4(M_0-M)}\, dM. \tag{3}$$

In the same way the partial emission coefficients can be defined for stars belonging to different physical types. Passage from numbers of stars in a unit volume or unit solid angle to the emission coefficient and the surface brightness corresponds to a passage from "microscopic" to "macroscopic" point of view.

The macroscopic approach has in fact been applied since long to the study of external galaxies for example to the study of the ellipticals (in particular in the work of Oort [5] on ellipticals NGC 3115 and 4494). In the galactic astronomy it was also used in a few cases, for example in the paper of Greenstein and Henyey [6] on the light scattered by cosmic dust and in author's studies [7,8] on fluctuations of surface brightness of the Milky Way.

In particular Greenstein and Henyey have obtained the formula (9) of this paper and done a comparison similar to that presented in Table 1 below. Howerever they have missed the opportunity to infer about the relative distribution of dark and bright matter.

Let us note that the value of the total emission coefficient in a vicinity of Sun can be successfuly deduced from the Van–Rhijn luminosity functionand equals 4.8 stars of absolute magnitude 10.0 in photographic system and to 6.1 stars of the same absolute magnitude in the visual part of the spectrum. This means that the colour-index of the mean radiation in the vicinity of the Sun is about +0.26.

§4. **The coefficient of absorption and the optical distance from the plane of Galaxy.** In the following discussion we distract from the patchy structure of absorbing medium in Galaxy and consider some macroscopic coefficient of absorbtion α assumed to be a continuous function of coordinates. For any point in Galaxy we form an integral over values of α along the perpendicular to the equatorial plane of Galaxy:

$$\tau = \int_0^z \alpha \, dz, \tag{4}$$

where z is the distance of the point from the equatorial plane. We call τ the optical distance from the equatorial plane. When z is increasing indefinitely, τ is approaching τ_0, the halfed total optical thickness of the absorbing layer:

$$\tau_0 = \int_0^\infty \alpha \, dz. \tag{5}$$

Of course τ is a function of x, y and z and τ_0 is a function of x and y only. In the approximation of plane-parallel layers (in the vicinity of the Sun) we consider τ_0 to be a constant. In the same approximation η is a function of z and therefore can be considered as a function of τ.

The value of τ_0 for photographic part of spektrum is $0^m.25$ according to Hubble [9] and is $0^m.34$ according to Parenago [10].

§5. **The total photographic brightness in the case of plane-parallel layers.** As we know from the work of Vashakidze [11] and Oort [12] the model of plane-parallel layers is in satiafactory agreement with the observed distribution of stars by apparent magnitudes for high galactic latitudes. Although deeper study reveals some deviations from such model we will use it as a satisfactory approximation. In this model τ and α are functions of τ and so is their ratio

$$\frac{\eta}{\alpha} = B(\tau).$$

The surface brightness at the galactic latitude b in this model will be

$$I(b) = \int_0^{\tau_0} \exp\left(-\frac{\tau}{\sin b}\right) B(\tau) \frac{d\tau}{\sin b} \tag{6}$$

This formula corresponds to the case where the stars are immersed in an absorbing layer. Assume a fraction of stars lies far enough from equatorial plane to fall out of the absorbing layer. Then instead of (6) we will have

$$I(b) = \int_0^{\tau_0} \exp\left(-\frac{\tau}{\sin b}\right) B(\tau) \frac{d\tau}{\sin b} + C \exp\left(-\frac{\tau_0}{\sin b}\right) \frac{1}{\sin b}. \tag{7}$$

The second term in this expression corresponds to the radiation of stars outside the absorbing layer.

Let us dwell on the case of formula (6) i.e. suppose that all stars are immersed in the absorbing layer. This will be true in particular if τ/α tends to zero as τ increases.

Let us accept that

$$B(\tau) = B_0 = \text{const} \quad \text{for} \quad \tau < \tau_1 \leq \tau_0 \tag{8a}$$

and

$$B(\tau) = 0 \quad \text{for} \quad \tau_1 < \tau \leq \tau_0, \tag{8b}$$

where τ_1 is a constant. This means that we assume that the ratio emission /absorption is constant within some distance from galactic plane and that on larger distances the emission coefficient is zero.

Then from (6) we have

$$I(b) = B_0 \left(1 - e^{-\tau_1 \operatorname{cosec} b}\right). \tag{9}$$

It is clear that B_0 is the brightness of the galactic equator and can be determined from observations. The question arises:

Is it possible to find a value of τ_1 yielding agreement between the surface brightnesses calculated from (9) and values calculated on the basis of Van–Rhijn data on the mean values of A_n?

The second column of the Table 1 gives some values of $I(b)$, calculated according to (9), assuming that $B_0 = 220$ stars of tenth magnitude (photographic) from a square degree and $\tau_1 = 0.12$. In the third column of the same table the values of I obtained via real counts are given.

b	I_c	I_{obs}
0°	220	220
10°	108	104
20°	65	66
30°	46	47
40°	37	37
90°	26	25

Table 1.

Comparing the two columns, we see that our model is in approximate agreement with the average data taken from observations. Thus the stars outside the absorbing layer do not influence the distribution of brightness in question. Thus the second term in (7) can be neglected. The optical thickness in (9) turns to be $\tau_1 = 0.12$. This is the halfed optical thickness of the layer in which the stars are immersed.

On the other hand since the minimal value of τ_0 is 0.23 or 0.25 (if expressed in stellar magnitudes), no doubt remains that the considerable part of the absorbing layer belongs to the region where the stars are rare. In other words, η/α tends to zero as the distance from the equatorial plane decreases.

Some other astronomers also came to the same conclusion for example Aller and Trumpler [13]. Oort [14] has noticed that the selectively absorbing matter tends to concentrate near the equatorial plane of Galaxy, while the matter causing neutral absorption is more dispersed.

§6. **More exact interpretation.** After this confirmation of applicability of the model of plane-parallel layers, we concentrate on the interpretation of numerical values of B_0 and τ_1.

Let us notice that for smaller values of τ_1, the formula (9) for the high galactic latitudes reduces to

$$I(b) = B_0 \tau_0 \operatorname{cosec} b \tag{10}$$

yielding

$$I\left(\frac{\pi}{2}\right) = B_0 \tau_0; \quad I(0) = B_0. \tag{11}$$

On the other hand, for $b = \frac{\pi}{2}$ and small τ_1, formula (6) yields

$$I\left(\frac{\pi}{2}\right) = \int_0^{\tau_0} B(\tau)\, d\tau \tag{12}$$

and for $b = 0$

$$I(0) = B(0). \tag{13}$$

From (12) and (13) we get

$$\frac{I\left(\frac{\pi}{2}\right)}{I(0)} = \frac{\int_0^{\tau_0} B(\tau)\, d\tau}{B(0)}$$

and from (10) and (11)

$$\tau_1 = \frac{I\left(\frac{\pi}{2}\right)}{I(0)}. \tag{14}$$

Comparing with the preceding formulas we obtain

$$\tau_1 = \frac{\int_0^{\tau_0} B(\tau)\, d\tau}{B(0)} \tag{15}$$

and taking into account that

$$\int_0^{\tau_1} B(\tau)\, d\tau = \int_0^{\infty} \eta\, dz \quad \text{and} \quad B(0) = \frac{\eta(0)}{\alpha(0)} \tag{16}$$

we find

$$\tau_1 = \alpha(0) \frac{\int_0^{\infty} \eta\, dz}{\eta(0)} = \alpha(0)\, z_1. \tag{17}$$

Thus τ_1 represents the optical thickness of a layer, whose coefficient of absorption equals that of the galactic plane and which has linear thickness

$$z_1 = \frac{\int_0^\infty \eta\, dz}{\eta(0)}. \tag{18}$$

In its turn this quantity is the halfed perpendicular linear thickness of homogeneous emitting Galaxy in the region around the Sun.

From (17) one can determine z_1, if the value of $\alpha(0)$ is known.

§7. **Determination of $\alpha(0)$.** From (13) and (16) we have immediately

$$I(0) = \frac{\eta(0)}{\alpha(0)}. \tag{19}$$

From (19) we can find $\alpha(0)$ since $I(0)$ is known from observations and $\eta(0)$ can be found from (3) because the luminosity functionin the vicinity of the Sun is also known.

We have already stated that $I(0)$ is equal 220 stars of the tenth apparent magnitude per square degree. Multiplying by 33, we express the same quantity in the stars of the tenth absolute magnitude per square parsec. We obtain $7.3 \cdot 10^3$ stars of the tenth absolute photographic magnitude from a square parsec. Using the luminosity functionof Van–Rhijn we find that $\eta(0)$ is equal to 0.048 stars of tenth absolute photographic magnitude from one cubic parsec. This amounts to $\alpha = 0.66$ per kiloparsec.

But this is the value of the absorbtion coefficient determined in the usual way. The same coefficient expressed in stellar magnitudes is 0.72 per kiloparsec.

Unfortunately we have no ideas about the accuracy of this value. This is connected in the first line with errors in determination of $\eta(0)$. The assumption of plane-parallel layers is another source of error. It is possible that the value of $\alpha(0)$ also requires some correction, particularly if the Sun lies in a relatively less populated region of galactic equatorial plane.

Because for the calculation of $I(0)$ we used the numbers of stars averaged over longitudes and since $I(0) = \frac{\eta(0)}{\alpha(0)}$, our value of $\alpha(0)$ is rather some harmonic mean. There can be considerable fluctuations of real value of $\alpha(0)$ depending on direction (longitude). Because of great fluctuations, the harmonic mean will essentially be smaller than arithmetic mean. Thus the arithmetical average must be larger than $0^m.72$ per kiloparsec. By a preliminary estimate, the difference can reach $0^m.10$. Therefore we evaluate the mean value of $\alpha(0)$ to be

$$\alpha(0) = 0.82 \frac{\text{stellar magnitude}}{\text{kiloparsec}}$$

All these estimates should be considered as preliminary.

§8. **Determination of z_1.** From (17) we have

$$z_1 = \frac{\tau_1}{\alpha(0)}.$$

Using the values of τ_1 and $\alpha(0)$ obtained above we get

$$z_1 = 160 \text{ parsec}.$$

This means that the total thickness of luminous Galaxy in the vicinity of Sun must be 320 parsec.

§9. **Determination of τ_1 for different types of stars.** Using the partial surface brightnesses for different classes of stars we can find the corresponding values of τ_1 as we did above for the ensemble of all stars.

However owing to scarcity of data on the spectral type or other parameters of faint stars we are able to receive more or less real data on the partial surface brightnesses only for such classes for which the surface brightness is determined mainly by stars of higher apparent luminosity.

To such classes belong for example B-stars and Cepheids. We determined τ_1 for them in the following way. For higher galactic latitudes we have from (9)

$$I(b) = B_0 \tau_1 \operatorname{cosec} b$$

and for the galactic equator

$$I(0) = B_0.$$

Therefore

$$\tau_1 = \frac{I(b)}{I(0)} \sin b \tag{20}$$

and in particular

$$\tau_1 = \frac{I(\frac{\pi}{2})}{I(0)}. \tag{20$'$}$$

Unfortunately for the classes of stars under consideration the number of stars in the high galactic latitudes is small. Therefore it is necessary to take sufficiently large circumpolar zone (for example from $\pm 30°$ to $\pm 90°$). In such a zone the latitude b changes considerably. Therefore we have to integrate over some solid angle ω. We write

$$\tau_1 = \frac{1}{\omega I(0)} \int I(b) \sin b \, d\omega. \tag{21}$$

Since the brightness under consideration consists of the contributions of separate stars, the integral reduces to a sum

$$\tau_1 = \frac{1}{\omega I(0)} \sum i \sin b, \tag{22}$$

where i is the brightness of individual star, the summation is over all stars of the given class in the region ω.

Comparing (20') and (22) we find

$$I\left(\frac{\pi}{2}\right) = \frac{1}{\omega} \sum i \sin b. \tag{23}$$

From (23), $I(\frac{\pi}{2})$ can be found even in case where there are (almost) no stars of the class in question in the small circumpolar region.

As regards $I(0)$, this quantity can be determined by summation of brightnesses of stars in some narrow equatorial region, for example between $\pm 5^\circ$ of galactic latitude.

Calculations based on some published lists of photographic brightnesses gave the following results.

a) The average partial visual surface brightness for stars $O-B_2$ near $b=0^\circ$ is 2.7 stars of tenth magnitude per square degree.

b) the average partial surface brightness of the totality of stars $O-B_2$ for $b=90^\circ$ is 0.23 stars of tenth magnitude per square degree.

These two conclusions are based on the data of HDC, which we consider as complete, since the B stars fainter than 8.0 give no essential contribution to the surface brightness.

From (22) and from numerical data cited above, for the stars $O-B_2$ we have

$$\tau_1 = 0.1.$$

However it is necessary to indicate a possible source of errors in such estimates. Since the number of such stars in high galactic latitudes is small, large relative fluctuations in the estimate of $I(\frac{\pi}{2})$ according to (23) are possible. In other words, occasional presence or absence near the Sun of a single star of the class in question can lead to quite different values of surface brightness.

Thus in our case the star α Virginis adds to (23) almost as much as all remaining 26 stars of $O-B_2$ type, for which $|b|>20$. Therefore we decided to adopt for $O-B_2$ stars the value $\tau_1 = 0.07$.

c) The average partial photographic surface brightness due to cepheids (classical) at $b=0^\circ$ is 0.14 stars of tenth magnitude per square degree.

d) The average partial photographich surface brightness due to classical cepheids at $b=90^\circ$ is 0.006 stars of tenth magnitude per square degree.

From the last two estimates we find $\tau_1 = 0.04$.

Unfortunately, the star δ Cephei is responsible for the considerable part of the brightness for equatorial zone. If we disregard this star we obtain for this zone $\tau_1 = 0.07$ and the mean value becomes $\tau_1 = 0.06$.

Thus we can adopt

$$\begin{array}{ll} \text{for} \quad O-B_2 & \tau_1 = 0.07 \\ \text{for cehpeids} & \tau_1 = 0.06. \end{array}$$

This result suggests strong concentration of stars of both type near the galactic plane within the absorbing layer.

§10. **The planetary nebulae.** The study of the distribution of planetaries on the sky brings to rather unexpected results. We have calculated the sum of the photographic brightnesses for different galactic belts. From these we deduced the average surface brightnesses now in the second column of Table 2.

b	I_{obs}	I_{c}
0°	44	40
7°	22	25
12°	46	30
20°	15	29
32°	15	25
60°	21	18

Table 2.

Compiling this table we used data from the list of planetary nebulae from the book of Vorontsov–Veljaminov "Novae and planetary nebulae". It is clear that the presence of a secondary minimum in $I(b)$ at $b = 7°$ contradicts formula (9). Therefore in formula (7) we assume that $B(\tau) = B(0)$ in the interval $0 \leq \tau \leq 0.04$, otherwise $B(\tau) = 0$. On the other hand we have taken that $C \neq 0$ due to considerable number of planetaries outside the absorbing layer.

This corresponds to the idea that there are two groups of planetaries: one inside $2r_1$ layer, while the other is widely dispersed with majority falling outside the absorbing layer.

For this case

$$I = B_0 \left(1 - \exp - \frac{\tau_1}{\sin b}\right) + C \exp - \frac{\tau_1}{\sin b}. \tag{24}$$

The table 2 contains values of I calculated for optimal values of constants:

$B_0 = 40$ stars of 20-th magnitude per square degree.

$C = 20$ stars of 20-th magnitude per square degree.

$\tau_1 = 0.04; \quad \tau_0 = 0.3$

We see that in spite of optimal choice of 3 constants the results are not good enough as compared with the case of all stars (Table 1).

Still a part of discrepancy can be due to small number of objects counted.

§11. **Colour of the Galaxy.** Imagine an observer situated outside our Galaxy far in the direction of its axis of rotation. What surface brightness he would observe in the region of the Sun? The answer is well known to be the doubled surface brightness which we observe in the direction of galactic pole, the interstellar absorption neglected. Since according to the existing data the colour of spiral arms is essentially different from the colour of other parts of the same stellar system, this indicator will be essential for deciding in which part of the Galaxy our Sun is situated.

The colour of the full stellar radiation from the poles of Galaxy can be deduced from statistics of colours of stars in that regions. However the data in this respect which exist are incomplete and not exact.

Still, if the data available allows to estimate merely the lower bound of the colour–index, this will be of some interest. In particular, if we can show that for the poles this index exceedes > 0.50 then the problem will be solved since in the spiral arms $c < 0.50$. We will see that such an estimate is possible.

Let us write i_k for the partial visual surface brightness of the galactic pole, produced by stars of some k-th spectral type. The photographic surface brightness coming from these stars will be $i_k \cdot 10^{-0.4c_k}$ where c_k is the colour index of stars of the type. Evidently the colour index of the total radiation will be

$$c = -2.5 \lg \frac{\sum i_k \cdot 10^{-0.4c_k}}{\sum i_k}. \qquad (25)$$

To determine a lower bound we consider two possibilities: either 1) $c \geq 0.^m6$ or 2) $c < 0.^m6$. If the first assumption is true, we already have a lower bound for the colour index equal to 0.6. In the case of the second assumption the observational data yield another estimate of the lower bound. Assume we have some quantities i'_k, which are equal to i_k for the spectral types B, A and F and are smaller than i_k for late types (G, K, M). Evidently

$$\frac{\sum i'_k \cdot 10^{-0.4c_k}}{\sum i'_k} > \frac{\sum i_k \cdot 10^{-0.4c_k}}{\sum i_k},$$

since the weighted mean of the quantities $10^{-0.4c_k}$ determined by the left hand side of this equation is obtained by means of replacing the weights i_k for later spectral types by smaller weights. Therefore,

$$c_1 = -2.5 \lg \frac{\sum i'_k \cdot 10^{-0.4c_k}}{\sum i'_k} < -2.5 \lg \frac{\sum i_k \cdot 10^{-0.4c_k}}{\sum i_k} = c. \qquad (26)$$

will be the desired lower bound for c.

Using data of HDC, we find with sufficient accuracy the values of i_k for the types B, A, F since for the determination of total brightnesses of stars of these types in polar directions, this catalogue is good enough. Some small corrections one can do using the data of BSD.

For this purpose we have used the statistic of HDC stars compiled by Charlier (17) and counts of stars in BSD given in the introduction to the second volume of this catalogue.

As regards G, K, M spectra, HDC and even BSD can hardly be considered as complete. Therefore instead of i_k we will find some quantities i'_k which are smaller than i_k.

In this way we have received the following values of i'_k for different spectral types:

S_p	B	A	F	G	K	M
i'_k	1	4	5	12	7	0

in stars of 10–th magnitude per square degree.

From this we obtained using (26): $c_1 = 0^m.55$. Thus in the case of the first assumption the lower bound of c is 0.6 and in the second case it is 0.55, i.e. we conclude that

$$c > 0.55.$$

This suggests (albeit indirectly) that the Sun is situated somewhere between the spiral arms of the Galaxy. Calculating the value of c, we have used the normal colours of the stars of each spectral type, since the selective absorbtion in the direction of poles requires rather small corrections.

§12. **Comparison of partial surface brightnesses for different galaxies.** It can be interesting to compare the partial surface brightnesses caused by stars of some physical type in outer galaxies with the same parameter in our Galaxy, when they are observed from outside under the same angle to the axis of rotation. A comparison for one particular case have been published by the author in [18].

§13. **Conclusions.** The study of surface brightnesses in our Galaxy brings to a number of interesting results. The main conclusions are: a) The stronger concentration of stars and lower of absorbing matter near Galactic plane and b) The yellow colour of the region of our Galaxy in which the Sun is situated, if observed from outside the Galaxy.

All conclusions of this paper are of preliminary nature. Our aim was to attract attention to a new direction of study, capable to produce convincing results by rather simple methods.

R E F E R E N C E S

1.Fesenkov, Russian Astron. Journal, vol. 22, no. 271, 1945.

2. Van–Rhijn, Groningen Publ., no. 43, 1929.

3. Lindblad, Handbuch der Astrophysic, Bd. 5, 1933.

4. B. V. Kukarkin, Manuscript. (not published).

5. Oort, Ap. J., vol. 91, no. 273, 1940.

6. Henyey and Greenstein, Ap. J., vol. 93, no. 70, 1941.

7. V. Ambartzumian, Dokl. Akad. Nauk Arm. SSR, vol. 44, no. 244, 1944.

8. V. Ambartzumian, Dokl. Akad. Nauk Arm. SSR, vol. 1, no. 9, 1944.

9. Habble, Moont Wilson Contr., no. 485, 1944.

10. P. P. Parenaro, Astron. Journal, vol.17, no.1, 1940: vol. 22, no. 129, 1945.

11. Vaschakidze, Bull. Abast. Obs., vol. 1, no. 87, 1937.

12. Oort, Bull. Astr. Inst., Netherlands, vol. 8, no. 245, 1938.

13. Aller and Trumpler, PASP, vol. 50, no.275, 1938.

14. Oort, Ann. d'Astrophysique, vol. 1, no. 71, 1938.

15 Seyfest, Ap. J., vol. 91, no. 528, 1940.

16. Malmquist, Lund Med., no. 37, 1927.

17. Charlier, Lund Med., no. 34, 1926.
18. V. Ambartzumian, Dokl. Akad. Nauk Arm. SSR, vol. 3, no. 39, 1945.

Yerevan
Astronomical Observatory of the Academy
of Sciences of Armenia.