TO THE THEORY OF FLUCTUATIONS OF THE SURFACE BRIGHTNESS IN THE MILKY WAY

At present we can take as established that the absorbing layer in our Galaxy has a patchy structure i.e. it consists of a large number of absorbing clouds whose cumulative effect is responsible for the general cosmic absorbtion. The absorbing clouds which are nearer to us and have higher absorbing power appear to us as dark nebulae A large number of papers devoted to the study of dark nebulae have been published. However a statistical study of the whole complex of absorbing clouds, which sometimes have low absorbing power, is still missing.

In an earlier paper [1], the author has shown that the fluctuations in the number of the extra-galactic nebulae are at least partly caused by patchy structure of the galactic absorbing layer. In [1] some mean value of optical thickness of a cloud was deduced. It turned to be equal to $0^m.27$.

We can adopt as a working hypothesis that the fluctuations of surface brightness along the Milky Way are also caused by absorbing clouds. Apparently such an assumption can be considered as the first approximation to the real situation. Then the question arises about the distribution law of the brightness fluctuations. In the next paragraphs we derive a differential equation which determines the distribution function. From the latter we find the values of moments of all orders.

1. Let us suppose that the equatorial plane of the Galaxy is homogeneously filled by stars and absorbing clouds up to the infinite distance.

This model assumption is not as bad as it seems, since the absorbtion at very large distances is almost complete and distant stars and clouds do not influence the observed brightnesses . At the same time we will assume that during passage of light through each cloud the same part of the intensity is absorbed. The transparence of a cloud we will denote by q.

If the number of clouds in some direction at distances less than s is $n(s)$ (this is of course a random function), then the light of a star placed at distance s will diminish by the factor $q^{n(s)}$. Assume an element dV of galactic space emits total energy $4\pi\eta dV$. Then the observed brightness in any direction within the galactic plane will be

$$\int_0^\infty q^{n(s)}\,\eta\,ds.$$

We are going to consider the distribution function of values of this integral:

$$f(I) = P\left(\int_0^\infty q^{n(s)}\,\eta\,ds > I\right).$$

For small values of $a > 0$ we can write

$$f(I) = P\left(\int_0^a q^{n(s)}\,\eta\,ds + q^{n(a)}\int_a^\infty q^{n(s)-n(a)}\eta\,ds > I\right). \tag{1}$$

Since a is small, the number $n(a)$ can take on only two values, $n(a) = 0$ and $n(a) = 1$. The probability of 0 is $1 - \nu a$ and of 1 is νa. Here ν is the mean number of clouds per unit lenght of light path. The probabilities of other values of $n(s)$ are small quantities of higher orders and can be neglected.

Correspondingly the integral $\int_0^a q^{n(s)}\,\eta\,ds$ can take on either a value ηa or $\eta\theta a$, where $0 < \theta < 1$. Therefore

$$\begin{aligned} f(I) =& (1-\nu a)\,P\left(\int_a^\infty q^{n(s)-n(a)}\eta\,ds > I - a\eta\right) + \\ &+ \nu a\,P\left(\int_0^\infty q^{n(s)-n(a)}\eta\,ds > \frac{I-\eta\theta a}{q}\right). \end{aligned} \tag{2}$$

But owing to homogeneous distribution of clouds in the space

$$P\left(\int_a^\infty q^{n(s)-n(a)}\eta\,ds > I\right) = P\left(\int_0^\infty q^{n(s)}\,\eta\,ds > I\right)$$

and the equation (2) can be rewritten in the form

$$f(I) = (1-\nu a)\,f(I-a\eta) + \nu a\,f\left(\frac{I-\eta\theta a}{q}\right). \tag{3}$$

Up to the terms of the second order in a, this is equivalent to

$$f(I) = f(I) - \nu a\,f(I) - a\eta\,f'(I) + \nu a\,f\left(\frac{I}{q}\right).$$

From this

$$f(I) + \frac{\eta}{\nu}\,f'(I) = f\left(\frac{I}{q}\right) \tag{4}$$

or using new variable $u = I\frac{\nu}{\eta}$

$$f(u) + f'(u) = f\left(\frac{u}{q}\right). \tag{5}$$

By differentiating, for the density $g(u) = f'(u)$ we obtain the equation

$$g(u) + g'(u) = \frac{1}{q}\,g\left(\frac{u}{q}\right). \tag{6}$$

From this functional equation it is possible to find the mean values of all powers of brightness.

From (5) we can see that

$$f'(0) = g(0) = 0.$$

Now multiplying (6) by u and integrating, we find

$$\overline{u} + \int_0^\infty g'(u)\,u\,du = q\overline{u},$$

where $\overline{u}$ is the mean brightness.

Integrating by parts and taking into account that

$$\int_0^\infty g(u)\,du = 1,$$

we find

$$\overline{u} = \frac{1}{1-q}. \tag{7}$$

Multiplying (6) by u^2 and integrating we get

$$\overline{u^2}\,(1-q^2) = -\int_0^\infty g'(u)\,u^2\,du = 2\overline{u},$$

i.e. the mean value of the squared brightness equals

$$\overline{u^2} = \frac{2\overline{u}}{1-q^2} = \frac{1}{(1-q^2)(1+q)}.$$

For the relative square deviation we obtain

$$\frac{\overline{(u-\overline{u})^2}}{\overline{u}^2} = \frac{\overline{u^2}}{\overline{u}^2} - 1 = \frac{1-q}{1+q}, \tag{8}$$

i.e. it is completely determined by transparency of one cloud.

2. Now we give up the assumption that all absorbing clouds have the same optical thickness. Rather let they have random optical thickness. However we suppose that in different parts of space the distribution of transparence q remains the same. Let the probability to have the value of transparence between q and $q + dq$ be $d\varphi(q)$.

In this case for the distribution function f of brightness we obtain the following generalization of (5)

$$f(u) + f'(u) = \int_0^1 f(u/q)\,d\varphi(q). \tag{9}$$

For the probability density $g(u) = -f'(u)$ we get

$$g(u) + g'(u) = \int_0^1 g\left(\frac{u}{q}\right)\frac{d\varphi(q)}{q}. \tag{10}$$

The moments $\overline{u}$ and $\overline{u^2}$ are found to be

$$\overline{u} = \frac{1}{1-\int_0^1 q\,d\varphi(q)} = \frac{1}{1-\overline{q}}; \qquad \overline{u^2} = \frac{2\overline{u}}{1-\int_0^1 q^2\,d\varphi(q)} = \frac{2\overline{u}}{1-\overline{q^2}}. \tag{11}$$

where $\overline{q}$ and $\overline{q^2}$ are the mean values

$$\overline{q} = \int_0^1 q\,d\varphi(q); \qquad \overline{q^2} = \int_0^1 q^2\,d\varphi(q).$$

Since $\overline{q^2} > \overline{q}^2$, we have

$$\overline{u^2} > \frac{2\overline{u}}{1 - \overline{q}^2}.$$

It turns out that given $\overline{q}$, the relative mean square deviation

$$\frac{\overline{u^2} - \overline{u}^2}{\overline{u}^2} = \frac{\int_0^1 (1-q)^2 \, d\varphi(q)}{\int_0^1 (1-q^2) \, d\varphi(q)}$$

is minimal in the case of clouds of identical transparence equal to $\overline{q}$.

For a number of simpler laws $\varphi(q)$, solutions of the equation (10) can be expressed by means of integrals.

Thus, if $\varphi(q) = q$ (uniform distribution of transparence in $(0,1)$)

$$g(u) = \int_{-\infty}^{\infty} e^{iut} \frac{dt}{(1+it)^2}. \tag{12}$$

If $\varphi(q) = q^2$, we have $\overline{q} = 2/3$ and

$$g(u) = \int_{-\infty}^{\infty} e^{iut} \frac{dt}{(1+it)^3}. \tag{13}$$

The above results give some idea on the dependence of the fluctuations of the Milky Way brightnesses on the parameters describing the set of absorbing clouds.

Note. It is easy to find that in the case of $\varphi(q) = q^k$ the equation (10) has a solution $g(u) = u^k \, e^{-u}$.

R E F E R E N C E S

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